

ANALYSIS OF INTERMEDIATE STRUCTURE
IN FISSION CROSS SECTION OF ^{241}Pu

Yasuyuki Kikuchi



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ABSTRACT

The intermediate structure observed in recent fission-cross-section data is analyzed by assuming a double-humped fission potential. From channel theory, the fission width of the 3^+ state is considered to have this structure.

The s-wave strength function deduced from resolved resonance analysis ($s_0 = 1.3 \times 10^{-3}$) is found to be too small to describe the peaks of the structure. The value from total-cross-section measurements ($s_0 = 1.9 \times 10^{-3}$) is used in this analysis. Therefore the fission width obtained is smaller and the α value obtained is larger than those from the ENDF/B Version II file.

The mean values of level spacing and width of the Class II states are 270 and 70 eV, respectively. From these values, the fission potential shape is estimated.

I. INTRODUCTION

Recent fission-cross-section data of ^{241}Pu show an intermediate structure similar to that in ^{239}Pu . This intermediate structure is expected because of the effect of the double-humped fission potential, since the structure of ^{239}Pu can be explained very well with this assumption.¹

In the ENDF/B Version II, this structure is almost neglected. The values of the fission width are obtained in the ENDF/B file² at discrete energy points from fission-cross-section data (the Petrel data)³ and the assumed capture-cross-section value by assuming that the ratios of $\Gamma_n^{(2+)}/\Gamma_n^{(3+)}$ and $\Gamma_f^{(2+)}/\Gamma_f^{(3+)}$ are constant for all energy points. This assumption concerning the fission width has no physical basis, however.

The purpose of this report is to analyze the Petrel data with the formulation by Weigmann,⁴ and to express the structure of the fission width with intermediate resonance parameters.

The same procedure¹ as used for the analysis of ^{239}Pu was employed and is only briefly described here.

II. ANALYSIS OF THE INTERMEDIATE STRUCTURE

A. Intermediate Structure of the Fission Cross Section

The fission-cross-section data are averaged over several energy intervals. With an interval of less than 50 eV, much fine structure is observed to be overlapped with a broad structure. The average amplitude of this fine structure is smaller than the statistical uncertainty calculated with the Monte Carlo method.

On the other hand, with intervals of more than 75 eV but less than 150 eV, most of the fine structure disappears and the broad structure remains. Since the average amplitude of this broad structure is more than the standard deviation due to the small sampling, this structure can be considered as intermediate structure due to the double-humped potential.

In the following analysis the cross-section data were averaged over various 100-eV intervals whose energy ranges overlap each other (0-100, 50-150, 100-200, ... eV).

B. Assumption on Fission Width from Channel Theory

For ^{239}Pu , the 1^+ state is known to have only one subthreshold channel, which is considered to have the intermediate structure. It is not clear for ^{241}Pu whether the 2^+ or the 3^+ state has the intermediate structure. Kikuchi and An⁵ estimated the spin-dependent fission width from channel theory, by assuming similar transition states for ^{233}U and ^{241}Pu . According to their analysis, the first channel of the 2^+ state corresponds to a ground-state band and is fully open. The second channel of the 2^+ state is a gamma vibration band, which also gives the first channel of the 3^+ state.

It is not clear to what extent this gamma vibration channel is open. However, we assume it to be partially open, since the intermediate structure is predominant for a subthreshold channel. We also neglect the intermediate structure of the 2^+ state, because the intermediate structure due to the second channel must be obscured by the fully open first channel.

We then get the following relation of the fission width averaged over many intermediate states:

$$\left. \begin{aligned} \bar{\Gamma}_f^{(2^+)} &= \frac{D^{(2^+)}}{2\pi} (1 + P); \\ \bar{\Gamma}_f^{(3^+)} &= \frac{D^{(3^+)}}{2\pi} P; \end{aligned} \right\} \quad (1)$$

where $D^{(J^\pi)}$ is the mean level spacing and P is the penetrability of the gamma vibration channel. The value of P is determined from fission-cross-section data averaged over many intermediate states.

C. Other Resonance Parameters

Other resonance parameters are for the most part taken from the ENDF/B Version II compilation. However, the s-wave strength function and the fission widths of the states excited with p-wave neutrons are modified.

The fission widths of the p-wave states in the ENDF/B file are due to the evaluation by Yiftah *et al.*⁶ and are determined very arbitrarily. We use the spin-dependent values estimated from channel theory by Kikuchi and An.⁵

The ENDF/B neutron widths cannot be used, as they depend on the ENDF/B fission width. The s-wave strength function was evaluated to be $1.3 \times 10^{-3} (\text{eV})^{1/2}$ by Yiftah *et al.*⁶ This value was deduced from the resolved resonance parameters below 62 eV. On the other hand, Craig and Westcott⁷ reported $s_0 = 1.9 \times 10^{-3} (\text{eV})^{1/2}$ from the total-cross-section measurement below 1 keV.

It was found impossible to express the peak of the intermediate structure with the relation expressed as Eq. 1 and the low value of the s-wave strength function ($s_0 = 1.3 \times 10^{-3}$) evaluated by Yiftah *et al.*⁶ If we use the high value of the s-wave strength function ($s_0 = 1.9 \times 10^{-3}$), the structure can be explained; hence, we used the high value. There is obviously ambiguity in this selection of the strength function which strongly affects the fission width in order to fit the same cross-section value.

With this s_0 value, the value of P in Eq. 1 is estimated to be 0.2; i.e., $\bar{\Gamma}_f^{(2^+)} = 445 \text{ meV}$.

All the resonance parameters used in the present analysis are tabulated in Table I.

TABLE I. The Resonance Parameters Used in the Calculation

J^π	D , eV	$\langle \Gamma_v \rangle$, meV	$\langle \Gamma_f \rangle$, meV	ν_f	$\langle \Gamma_n^0 \rangle$, $10^{-3} (\text{eV})^{1/2}$
2^+	2.33	30	445	1	0.498
3^+	1.59	30	-	1	0.275
1^-	2.98	30	720	2	0.745
2^-	2.33	30	200	1	1.164
3^-	1.59	30	450	2	0.796
4^-	1.76	30	170	1	0.440

D. Representation of Intermediate Resonance Parameters

The contributions from the states other than the 3^+ state are calculated with the resonance parameters in Table I and are subtracted from the Petrel data averaged over 100 eV. The standard deviation due to the small sampling is also calculated as in Ref. 1. The fission cross sections of the 3^+ state thus obtained, averaged over 100 eV, are shown as open circles with their standard deviations (vertical bars) in Fig. 1.

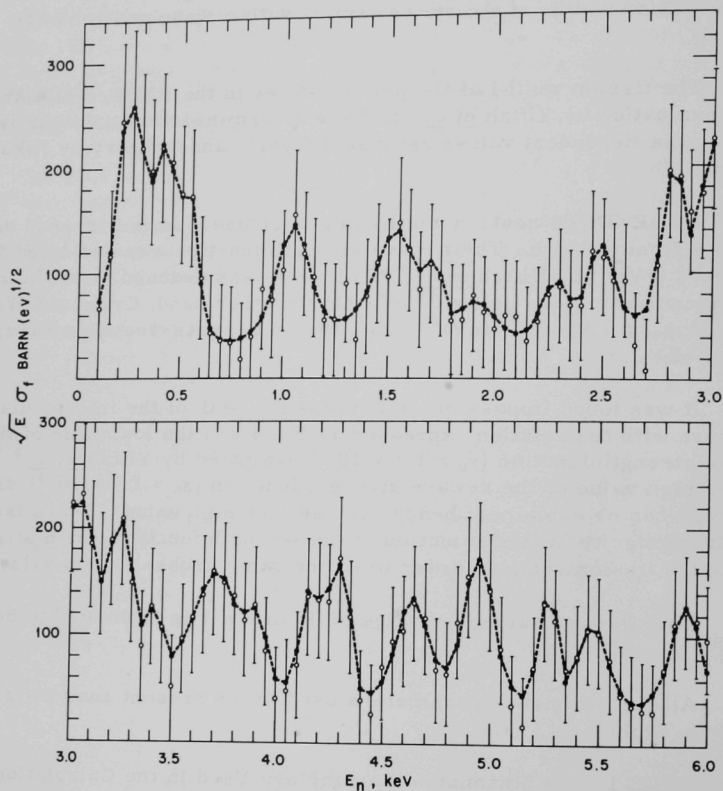


Fig. 1. Fission Cross Section of the 3^+ State Averaged over 100-eV Intervals Whose Energy Ranges Overlap Each Other. The points designated with open circles with vertical lines are deduced from the Petrel data. The data points designated with closed circles connected with a dashed line are calculated from the intermediate resonance parameters in Table II. ANL Neg. No. 116-202.

TABLE II. Intermediate Resonance Parameters

E_i , eV	Γ_i^\uparrow , eV	\overline{W}_i^2 , (eV) ²	Peak Value, eV
246	33	5.5	0.667
404	30	2.9	0.387
534	25	1.3	0.208
1050	109	1.8	0.066
1540	170	2.5	0.059
1707	11	0.3	0.109
1921	173	0.6	0.014
2280	95	0.62	0.026
2491	80	1.2	0.060
2806	10	2.1	0.840
3031	110	9.0	0.327
3247	22	3.1	0.564
3401	10	0.6	0.240
3771	182	4.5	0.099
3900	27	0.64	0.095
4164	32	1.3	0.163
4298	20	1.9	0.380
4641	102	2.1	0.082
4947	72	3.2	0.178
5260	18	1.1	0.244
5471	107	1.5	0.056
5903	106	2.1	0.079
Mean	70	2.3	0.129

The mean fission width at energy E can be expressed as^{1,4}

$$\langle \Gamma_f(E) \rangle = \sum_i \frac{\overline{W}_i^2 \Gamma_i^\uparrow}{(E - E_i)^2 + \frac{1}{4}(\Gamma_i^\uparrow)^2}, \quad (2)$$

where E_i , Γ_i^\uparrow , and \overline{W}_i^2 are the intermediate resonance parameters defined in Ref. 1. The least-squares fitting method is used to obtain the best set of the intermediate resonance parameters.

The best set of the intermediate resonance parameters so obtained are tabulated in Table II. The calculated fission cross sections are given in Fig. 1 as the closed circles connected with a dashed line. It is seen that the calculated values from the intermediate resonance parameters describe the structure in the Petrel data very well.

III. DISCUSSION

A. Comparison with the ENDF/B

A curve of the fission width calculated from the intermediate resonance parameters is compared with the point-wise data in the ENDF/B with the recommended interpolation law (linear-linear) in Fig. 2. The present calculated fission widths are much lower than those of ENDF/B, because of the assumed large strength function. It is also seen that the data in the ENDF/B have little structure.

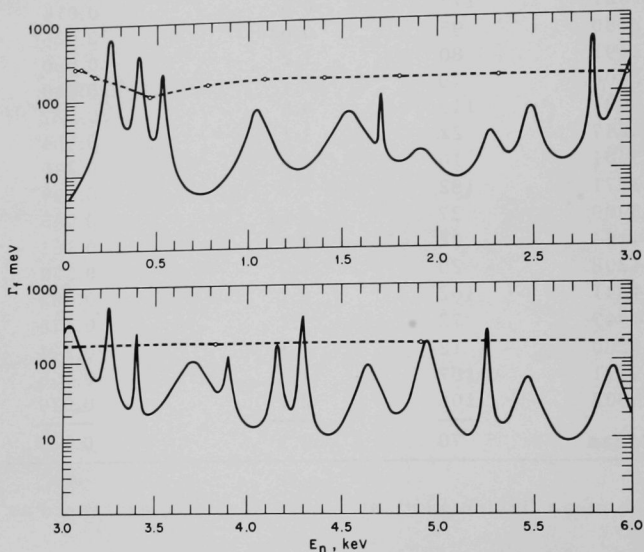


Fig. 2. Fission Width of the 3^+ State. The solid line is calculated from the intermediate resonance parameters. The points and the dashed line indicate the evaluated values in the ENDF/B file and the recommended interpolation (linear-linear). ANL Neg. No. 116-203 Corr.

Figure 3 compares the calculated fission cross sections with the Petrel data. The intermediate structure is very well described with the intermediate resonance parameters. The structure is fairly well described (below 500 eV) with the resonance parameters in the ENDF/B, but is neglected above 500 eV. The structure below 500 eV is considered as due to the structure both in the neutron width and in the fission width in the ENDF/B.

The calculated α values are compared with each other in Fig. 4. The α values obtained from the intermediate parameters are much higher than those from the ENDF/B file, because of the smallness of the present calculated fission width.

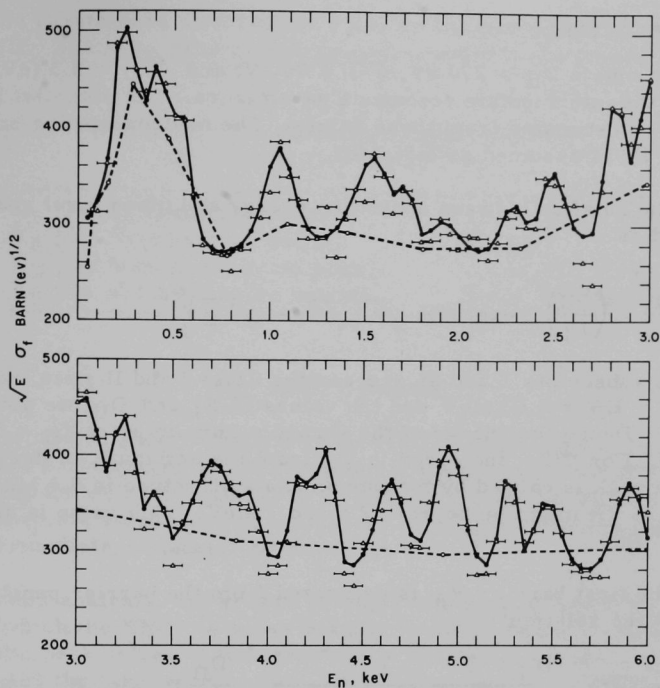


Fig. 3. Total Fission Cross Section. The points designated by triangles with horizontal lines are the Petrel data averaged over 100-eV intervals. The points designated with closed circles connected with a solid line are calculated from the intermediate resonance parameters. The data points designated by open circles connected with a dashed line are obtained from the ENDF/B compilation. ANL Neg. No. 116-207.

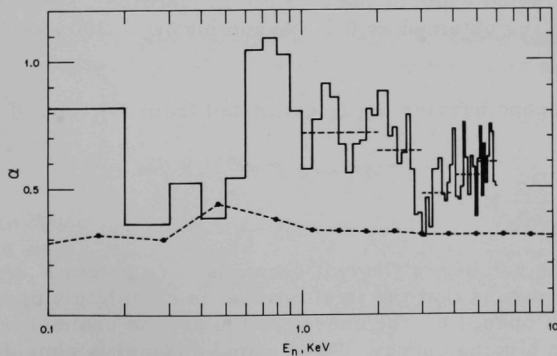


Fig. 4. Histogram of α Values. The histogram is calculated from the intermediate resonance parameters. The points connected with a dashed line are calculated from the ENDF/B file. ANL Neg. No. 116-206.

B. Potential Shape

We obtain $D_{II} = 270$ eV, $\langle \Gamma^\dagger \rangle = 70$ eV, and $\langle \bar{W}^2 \rangle = 2.3$ (eV)² as mean values of the intermediate resonance parameters. The potential height of ²⁴²Pu can be estimated from these values. The neutron binding energy (E_B) of ²⁴²Pu is assumed as 5.52 MeV.

The relation between excitation energy and mean level spacing is expressed as

$$\frac{D_{II}}{D_I} = \left(\frac{E_{II}}{E_I} \right)^{5/4} \exp \left[0.56 \sqrt{A} (\sqrt{E_I} - \sqrt{E_{II}}) \right], \quad (3)$$

where the subscripts I and II express the Class I and II state, respectively. From $E_I = E_B = 5.52$ MeV and the values of D_I and D_{II} , we obtain $E_{II} = 2.75$ MeV. Hence the energy of the second minimum point $V_{II} = E_I - E_{II} = 2.77$ MeV. For ²⁴⁰Pu, the error in V_{II} from the ambiguity of deciding whether a peak is caused by the intermediate structure is 0.4 MeV.¹ Hence the value of V_{II} might be between 2.6 and 3 MeV. This value is similar to that for ²⁴⁰Pu.

The first barrier V_A is estimated from the barrier penetrability P , which has the relation¹

$$\frac{2\pi}{D_I} \langle \bar{W}^2 \rangle = \frac{D_{II}}{2\pi} \frac{1}{1 + \exp \left[\frac{2\pi}{\hbar\omega} (V_A - E_B) \right]} = \frac{D_{II}}{2\pi} P, \quad (4)$$

where $\hbar\omega$ is a measure of the barrier curvature. From D_I , D_{II} , and $\langle \bar{W}^2 \rangle$, P is obtained as 0.21. This value is consistent with that obtained from the cross section averaged over many intermediate peaks in Section II.B. Hence the assumed fission width of the 2^+ state is justified. From $P = 0.21$, $(1/\hbar\omega)(V_A - E_B)$ is obtained as 0.2. Assuming $\hbar\omega = 300$ keV, $V_A - E_B \approx 60$ keV.

The second barrier V_B is estimated from D_{II} and $\langle \Gamma^\dagger \rangle$ with the relation

$$\langle \Gamma^\dagger \rangle = \frac{D_{II}}{2\pi} N, \quad (5)$$

where N is the number of the exit channels. We obtain $N = 1.6$ from $\langle \Gamma^\dagger \rangle$ and D_{II} . This means that the first channel is completely open and the second channel is half open; i.e., the energy of the second channel is nearly the same as the neutron binding energy. The second channel is considered to be the coupling of a mass asymmetry and a bending vibration,⁵ and the first channel to be a gamma vibration, as discussed in Section II.B. For ²⁴⁰Pu, the energy difference of these two channels is about 800 keV.⁵ Hence we can assume $E_B - V_B = 600$ to ~ 1000 keV.

The parameters obtained for a double-humped potential are tabulated in Table III. The zero point of energy corresponds to the ground state of ^{242}Pu .

TABLE III. The Estimated Parameters for the Fission Barrier of ^{242}Pu

E_B (neutron binding energy)	5.52 MeV
V_A (the first barrier height)	5.6 MeV
V_{II} (the second minimum point)	2.6 to ~ 3.0 MeV
V_B (the second barrier height)	4.5 to ~ 5.0 MeV

IV. CONCLUSIONS

The intermediate structure of the fission cross section of ^{241}Pu can be explained as being due to the structure of the fission width of the 3^+ state, by assuming the relation between $\Gamma_f^{(2^+)}$ and $\Gamma_f^{(3^+)}$ from the channel theory.⁵ The intermediate structure below 6 keV is very well explained with 22 sets of the intermediate resonance parameters.

In this analysis, the s-wave strength function evaluated by Yiftah *et al.*⁶ on which the ENDF/B value depends is shown to be too small to describe the peak values of the structure in the fission-cross-section data. We have used the large s-wave strength function obtained from the total-cross-section measurement by Craig and Westcott.⁷ This large strength function leads to large disagreements between the present results and those from the ENDF/B in the fission width and α value. We obtain a smaller fission width and a larger α value. Since no reliable experimental data are available for the total cross section and α value, it is impossible to decide which strength function is correct.

The shape of the fission potential barrier is estimated from the intermediate resonance parameters obtained here and is similar to that for ^{239}Pu .

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